

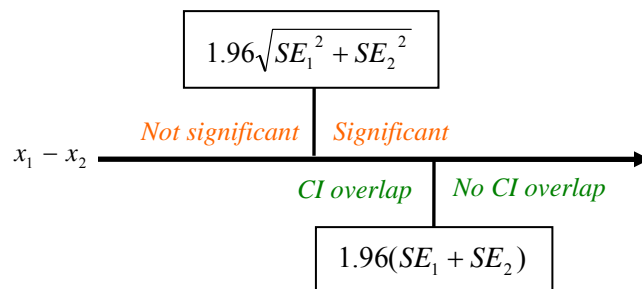
The root of the discrepancy is that distance from the mean is calculated in a different way for the t-statistic than it is for mean confidence intervals. Notice that mean confidence interval bounds are based on the magnitude of the standard errors (SE_1, SE_2) while the calculation for the test statistic for the difference between two means is based on the square root of the sum of squares of the standard errors ($\sqrt{SE_1^2 + SE_2^2}$).

Two means are significantly different ($\alpha = 0.05$) when $(x_1 - x_2) - 1.96\sqrt{SE_1^2 + SE_2^2} > 0$, that is, when the CI for the difference between the two group means does not contain zero. Two means do not have overlapping confidence intervals if $x_1 - 1.96SE_1 > x_2 + 1.96SE_2$, that is, if the lower bound of the CI for the greater mean is greater than the upper bound of the CI for the smaller mean. With a little bit of algebraic manipulation we find that:

The means are significantly different when: $x_1 - x_2 > 1.96\sqrt{SE_1^2 + SE_2^2}$

There is no overlap between CI when: $x_1 - x_2 > 1.96(SE_1 + SE_2)$

It is always the case that the square root of the sum of squares of two numbers is less than the sum of those numbers, $\sqrt{SE_1^2 + SE_2^2} < SE_1 + SE_2$. This creates a situation where as the difference in the means ($x_1 - x_2$) increases, it becomes significantly different before the two group mean confidence intervals cease to overlap.



The figure illustrates that there is a space where the difference in the means is significant and there is confidence interval overlap. To reiterate, notice that if we are in the CI overlap space, we cannot determine if there is a significant difference or not. If we know there is no CI overlap, we can be sure there is a significant difference.